

Grimes on the tacking by disjunction problem

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Abstract

In this paper, I focus on the so-called 'tacking by disjunction problem.' Namely, the problem to the effect that, if a hypothesis h is confirmed by a statement e , h is confirmed by the disjunction $e \vee f$, for whatever statement f . I show that the attempt to settle this difficulty made by Grimes 1990, in a paper apparently forgotten by today methodologists, is irremediably faulty.

A simple formulation of Hypothetico-deductivism states that, if h is a hypothesis and e an evidential statement:

(HD) e confirms h if (1) e is true, (2) h and $\sim e$ are each consistent, and (3) $h \vdash e$ [i.e. h entails e].¹ (Grimes 1990, 520).

(HD) implies the following rule:

(TDP) If e confirms h in accordance with (HD), then the disjunction $e \vee f$ will also confirm h where f is any arbitrary statement that is not entailed by $\sim e$. (*Ibid.*)

Consider in fact that, if e confirms h in accordance with (HD), h entails e , thus, by the rule of the introduction of disjunction, (3) h entails $e \vee f$ too. Moreover, if e confirms h in accordance with (HD) and f is not entailed by $\sim e$, (2) h and $\sim(e \vee f)$ prove each consistent. Thus, if (1) $e \vee f$ is true, in accordance with (HD), $e \vee f$ confirms h .

¹ Notice that, if (2) h and $\sim e$ are each consistent and (3) h entails e , then h and e must be *contingent*. This makes them liable to confirm and to be confirmed.

(TDP) is problematic, first of all, because it can make h 's confirmation depend on the verification of statements that are intuitively *irrelevant* for such a confirmation. Let us suppose, for example, that h is Newton's Mechanics, e is its consequence that a certain body A , if dropped, falls in accordance with the Free-Fall Law, and f is the statement 'my name is John.' In this case, since e confirms h in accordance with (HD) and f is not entailed by $\sim e$, on the grounds of (TDP), $e \vee f$ confirms h . Consider now the case in which e has *not* been verified and my name is in fact John. Because of the truth of f , $e \vee f$ is true too. Thus, given (TDP), Newton's Mechanics is confirmed by the fact that my name is John, which is hardly acceptable.

Indeed, there are even more counterintuitive cases of confirmation allowed by (TDP). Let us suppose that h – i.e. Newton's Mechanics – entails two statements e_1 and e_2 , where both of them confirm h in accordance with (HP). e_1 says again that the body A falls in accordance with the Free-Fall Law and e_2 says that another body B falls the same way. Consider now the disjunction $e_1 \vee \sim e_2$. Since $\sim e_2$ is not entailed by $\sim e_1$ and e_1 confirms h in accordance with (HD), given (TDP), the disjunction $e_1 \vee \sim e_2$ will confirm h too. This means that, if $\sim e_2$ is true, h is confirmed. But this is just absurd, as h entails the logical negation of $\sim e_2$.

Grimes believes that 'the basic idea underlying the [... hypothetico-deductive] method is that a hypothesis is confirmed if part of its content, part of what it asserts about the world, is shown to be true.' (Grimes 1990, 517). On the other hand, according to him, in the paradoxical confirmation cases allowed by (HD), 'though h will entail the disjunction $e \vee f$ if h entails e , the disjunction seems not to be part of the content of h .' (Grimes 1990, 520). Grimes is thus persuaded that the tacking by disjunction problem can be settled if (HD) is replaced by a rule closer to the basic hypothetico-deductivist idea that a hypothesis is confirmed if part of its content is shown to be true.

To articulate his solution, Grimes formulates Hypothetico-deductivism by resorting to the notion of a *disjunction in Boolean normal form* and the notion of a *narrow consequence*. As Grimes explains:

Each disjunct of a disjunction in Boolean normal form consists of a conjunction which includes in alphabetical order exactly one occurrence of each atomic expression or its negation (but not both) that occurs in the larger disjunction, where no two disjuncts are the same. For example, (p

$& q) \vee (\sim p \& q) \vee (\sim p \& \sim q)$ is a disjunction in Boolean normal form that is equivalent to the conditional $p \supset q$. (Grimes 1990, footnote 4, 520).

Grimes then defines the notion of a narrow consequence, represented by the symbol ' $\vdash\rightarrow$ ', as follows:

$p \vdash\rightarrow q \equiv_{df} p$ and $\sim q$ are each consistent and there is a disjunction $d_1 \vee \dots \vee d_k$ in Boolean normal form such that (i) $\vdash q \equiv (d_1 \vee \dots \vee d_k)$, and (ii) for some d_i , $p \vdash d_i$. (Grimes 1990, 520).

According to Grimes, 'the notion of a narrow consequence seems better suited [than that of a logical consequence] for expressing a relation of preserving content.' (Grimes 1990, 520).

Eventually, Grimes formulates Hypothetico-deductivism in terms of this principle:

(HD1) e confirms h if (1) e is true, (2) h and $\sim e$ are each consistent, and (3) $h \vdash\rightarrow e$.² (*Ibid.*)

Grimes' solution of the tacking by disjunction problem is *apparently* efficacious. Let us suppose, for instance, that d , e and f are atomic expressions, that $h \equiv_{df} d \& e$, and that h and $\sim (e \vee f)$ are each consistent. Notice that, in this case, the verification of f is surely irrelevant for h 's confirmation. Since h entails e and e entails $e \vee f$, h entails $e \vee f$ too. However, this disjunction is *not* a narrow consequence of h . For $e \vee f$ is equivalent to the disjunction in Boolean form $(e \& f) \vee (\sim e \& f) \vee (e \& \sim f)$, where h entails none of these disjuncts. Since the requirement (3) of (HD1) is not fulfilled, $e \vee f$ does not confirm h . More generally, Grimes' solution depends on the fact that the rule of the introduction of the disjunction does not hold on the notion of strict consequence. Thus, if $h \vdash\rightarrow e$, it is not true that, for every f , $h \vdash\rightarrow e \vee f$.

Grimes' proposal has surely the merit of being very intuitive, simple and elegant. If it worked, it could probably be preferred to other attempted solutions available in literature now, which are more complex.³ Unfortunately, Grimes' solution is faulty. The trouble with

² If (2) h and $\sim e$ are each consistent and (3) e is a strict consequence of h (thus h entails e), h and e are contingent and liable to confirm and to be confirmed.

³ See for instance Kuipers (2000, 17-27).

(HD1)⁴ is that, though it does not entail the problematic rule (TDP), it cannot dismiss at least some of the most counterintuitive confirmation cases, allowed by (TDP), that I have discussed above. (HD1) in fact entails that:

(TDP1) If e_1 and e_2 are atomic sentences that confirm h in accordance with (HD1), then the disjunction $e_1 \vee \sim e_2$ will also confirm h if $\sim e_2$ is not entailed by $\sim e_1$.⁵

(HD1) implies (TDP1) because $e_1 \vee \sim e_2$ is equivalent to the disjunction in Boolean normal form $(e_1 \& \sim e_2) \vee (\sim e_1 \& \sim e_2) \vee (e_1 \& e_2)$, where the last disjunct is *entailed* by h . For h implies, by assumption, both e_1 and e_2 .⁶

The problems affecting Grimes' solution are indeed more serious. For the notion of a narrow consequence of a theory is meant, by Grimes, to express 'a relation of preserving content', but it is evident that it does not do. The reason is that the disjunction $e_1 \vee \sim e_2$ and other statements one can construct at one's will, like $(\sim e_1 \& e_2) \vee (e_1 \& \sim e_2) \vee (e_1 \& e_2)$ and $(\sim e_1 \& \sim e_2) \vee (e_1 \& e_2)$,⁷ which are clearly *not* part of h 's content, prove all strict consequences of h . Notice finally that all these statements do confirm h in accordance with (HD1). Thus, for instance, on the grounds of $(\sim e_1 \& \sim e_2) \vee (e_1 \& e_2)$, whenever $\sim e_1 \& \sim e_2$ is verified, h is confirmed, which is completely unacceptable.

A seemingly promising strategy to settle such difficulties might be that of re-defining the notion of a strict consequence to the effect that q strictly follows from p if and only if both p and $\sim q$ are consistent and p entails *each* disjunct (individually taken) of the disjunction in Boolean normal form equivalent to q . Notice in fact that, if h entails e_1 and e_2 , h does not entail each disjunct of the disjunction in Boolean form

⁴ Unnoticed in the substantially positive review of Grimes' paper made by Skyrms 1992.

⁵ Probably, (TDP1) could be re-formulated to include couples of statements e_1 and e_2 that are not atomic. The present formulation is however sufficient to show the flaws in Grimes' solution.

⁶ More exactly, since h implies one of those disjuncts, (3) $e_1 \vee \sim e_2$ is a strict consequence of h . Moreover, since e_1 confirms h in accordance with (HD1) and $\sim e_2$ is not entailed by $\sim e_1$, (2) h and $\sim(e_1 \vee \sim e_2)$ prove each consistent. Thus, when (1) $e_1 \vee \sim e_2$ is true, in accordance with (HD1), $e_1 \vee \sim e_2$ confirms h .

⁷ Notice that all these statements are *already* disjunctions in Boolean normal form.

equivalent to $e_1 \vee \sim e_2$; the same happens in all other unwanted confirmation cases considered above. Unfortunately, this easy loophole does not work, as so defined a notion of a strict consequence is actually *too strict*. This notion appears to fit well intuitive content relations between simple logical conjunctions. For example, it is intuitive that $Pa \ \& \ Qa$ is part of the content of $\forall x(Px \ \& \ Qx)$ and, in this case, $\forall x(Px \ \& \ Qx)$ entails just each disjunct of the disjunction in Boolean form equivalent to $Pa \ \& \ Qa$. Consider however the following two statements that are not logical conjunctions: $\forall x(Px \supset Qx)$ and $Pa \supset Qa$. It is intuitive that $Pa \supset Qa$ is part of the content of $\forall x(Px \supset Qx)$, and yet the latter does *not* entail each disjunct of the disjunction in Boolean form equivalent to the former. Further examples similar to the latter can easily be produced.

I do not see any immediate ‘technical refinement’ of Grimes’ notion of a strict consequence or of (HD1) that could enable Grimes to overcome the difficulties I have discussed in this paper. My conclusion is that Grimes’ solution of the tacking by disjunction problem is seriously flawed and it cannot be accepted.

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